Integration

3.1 Integration as the Inverse of Differentiation, Integration of axn and integration of the Functions of the Sum/Difference of Algebraic Terms

$\int a\,dx = ax + C$

Example

$\int 2\,dx = 2x + C$

Exercise 3.1:

1. Find
   a. $\int 4\,dx$
   b. $\int 120\,dx$
   c. $\int -12\,dx$
   d. $\int \frac{2}{3}\,dx$
   e. $\int -\frac{1}{4}\,dx$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Example

1. $\int x^2 dx = \frac{x^3}{3} + C$

2. Find
   a. $\int x^4 dx =$
   b. $\int x\,dx =$
   c. $\int x^{-2} dx =$
   d. $\int x^{\frac{1}{2}} dx =$
   e. $\int x^{\frac{4}{5}} dx =$

Example

$\int \frac{1}{x^5} dx = \int x^{-5} dx = \frac{x^{-4}}{-4} + C$

   f. $\int \frac{1}{x^3} dx =$
   g. $\int \frac{1}{x^7} dx =$

Example

$$\int \sqrt{x}dx = \int x^{\frac{1}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2x^{\frac{3}{2}}}{3} + C$$

h. $\int \sqrt{x}dx =$

i. $\int \sqrt{x^3}dx =$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$

Example

$$\int 2x^3 dx = \frac{2x^4}{4} + C = \frac{x^4}{2} + C$$

3. Find

a. $\int 3x^5 dx =$

b. $\int -7x^3 dx =$

c. $\int -130x^{550}dx =$

d. $\int \frac{3}{4}x^2 dx =$

e. $\int -\frac{7}{15}x^6 dx =$

f. $\int \frac{5x^4}{9}dx =$

g. $\int -\frac{2x^5}{7}dx =$

h. $\int \frac{x^{-3}}{2} dx =$

i. $\int \frac{3x^{-2}}{5} dx =$

j. $\int \frac{4x^{\frac{1}{2}}}{11} dx =$

k. $\int \frac{2x^{\frac{2}{3}}}{3} dx =$

l. $\int -\frac{5}{7} x^{\frac{1}{3}} dx =$

m. $\int \frac{4}{7} x^{-\frac{3}{4}} dx =$

Example

$$\int \frac{2}{3x^5} dx = \int \frac{2}{3} x^{-5} dx = \frac{2}{3}\left(\frac{x^{-4}}{-4}\right) + C$$

$$= \frac{\cancel{2}}{3}\left(\frac{x^{-4}}{\cancel{-4}}\right) + C = \frac{x^{-4}}{-6} + C$$

n. $\int \frac{2}{x^2} dx =$

o. $\int \frac{7}{x^3} dx =$

p. $\int \frac{-2}{x^3} dx =$

Example

$\int \sqrt{2x} dx = \int \sqrt{2} x^{\frac{1}{2}} dx$

$= \sqrt{2}(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}) + C = \frac{2\sqrt{2}x^{\frac{3}{2}}}{3} + C$

t. $\int 2\sqrt{x} dx =$

q. $\int \frac{2}{5x^5} dx =$

u. $\int -7\sqrt{x^3} dx =$

r. $\int -\frac{3}{4x^7} dx =$

v. $\int \frac{2}{3}\sqrt{x} dx =$

s. $\int \frac{5}{7x^6} dx =$

w. $\int \sqrt{3x} dx =$

x. $\int \sqrt{4x^5}\,dx =$

y. $\int \sqrt[3]{6x^2}\,dx =$

z. $\int \sqrt[4]{2x}dx =$

$\int (u \pm v)dx = \int udx \pm \int vdx$

u and v are functions in x

Example

1. $\int 3x^2 + 2xdx = \int 3x^2dx + \int 2xdx$

$= \frac{3x^3}{3} + \frac{2x^2}{2} + C = \frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}} + C$

$= x^3 + x^2 + C$

4. Find

a. $\int x^3 + 2dx$

b. $\int x^2 + 2xdx$

c. $\int 2x^2 - 5x^3 + 1dx$

d. $\int 12x^7 - 8x^{-3}dx$

e. $\int \frac{1}{2}x^3 + 9x^8dx$

f. $\int \frac{1}{x^3} + \frac{2}{x^4} dx$

Example

$$\int (x+2)(3x+1)dx = \int 3x^2 + 7x + 2dx$$
$$= \int 3x^2 dx + \int 7x dx + \int 2dx$$
$$= \frac{3x^3}{3} + \frac{7x^2}{2} + 2x + C$$
$$= x^3 + \frac{7x^2}{2} + 2x + C$$

i. $\int (2x+3)^2 dx$

g. $\int \frac{1}{2x^3} + \frac{9}{2}x^8 dx$

j. $\int x^2(5x+3)dx$

h. $\int \frac{2}{3x^5} + 9x^{\frac{1}{2}} + 3dx$

k. $\int (3x+1)(x^2-2)dx$

l. $\int x(x^3+\frac{3}{x})dx$

Example

$$\int \frac{3x^3+x^2-x}{x}dx = \int 3x^2+x-1dx$$
$$= \int 3x^2dx + \int xdx - \int 1dx$$
$$= \frac{3x^3}{3} + \frac{x^2}{2} - x + C$$
$$= x^3 + \frac{x^2}{2} - x + C$$

o. $\int \frac{x^2-3x}{x}dx$

m. $\int (x^2+5)^2 dx$

p. $\int \frac{x^5-3x^3+5}{x^2}dx$

n. $\int (x+1)(x^2+3x-2)dx$

q. $\int \frac{(2x-1)^2}{x}dx$

r. $\int \frac{(2x^3-2)(x+5)}{x^2}dx$

s. $\int \frac{(x^2-3x)(x+5)}{x^3}dx$

3.2 Integration by Substitution

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

Example

$$\int (3x+5)^3 dx = \frac{(3x+5)^4}{3(4)} + C$$

$$= \frac{(3x+5)^4}{12} + C$$

Exercise 3.2:

1. Find
   a. $\int (x-1)^5 dx$

   b. $\int (5x-9)^3 dx$

   c. $\int (12x+3)^7 dx$

d. $\int (2x-18)^{-2}\,dx$

e. $\int (7x+6)^{\frac{5}{2}}\,dx$

f. $\int \frac{1}{(2x-3)^2}\,dx$

g. $\int \frac{2}{(5x+7)^3}\,dx$

h. $\int \frac{2}{3(9x-2)^5}\,dx$

i. $\int \sqrt{2x+5}\,dx$

j. $\int \sqrt{9x-4}\,dx$