Differentiation Exercise II

Sum and Difference Rule

$y = u \pm v$ u and v are functions in x

$$\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

Example

$y = 2x^3 + 5x^2$

$$\frac{dy}{dx} = 2(3)x^2 + 5(2)x = 6x^2 + 10x$$

Exercise 1

Find $\frac{dy}{dx}$ of the following

1. $y = 3x^2 - x$

2. $y = 120x^3 - 21x + 2$

3. $y = -3x^{\frac{2}{3}} - \frac{2}{3}x - 1$

4. $y = 9x^4 - 12x^3 - 2$

5. $y = 2x - \frac{2}{x}$

6. $y = \frac{2}{5}x - \frac{2}{x^2}$

7. $y = \frac{3}{5x^3} - \frac{1}{x^5}$

8. $y = \frac{3x^3}{2} - \frac{1}{2x^2}$

9. $y = \frac{3x^3 - 2x + 4}{x}$

10. $y = \frac{5x^3 - 7x + 4}{2x^2}$

Chain Rule

$y = u^n$ u and v are functions in x

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Example

$y = (2x^2 + 3)^5$

$u = 2x^2 + 3$, therefore $\frac{du}{dx} = 4x$

$y = u^5$, therefore $\frac{dy}{du} = 5u^4$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 5u^4 \times 4x$

$= 5(2x^2 + 3)^4 \times 4x = 20x(2x^2 + 3)^4$

Or differentiate directly

$y = (2x^2 + 3)^5$

$\frac{dy}{dx} = 5(2x^2 + 3)^4 \times 4x = 20x(2x^2 + 3)^4$

Exercise 2

Differentiate the following equation upon x.

1. $y = (2x + 3)^3$

2. $y = (2x^2 + 3x + 3)^2$

3. $y = (\frac{2}{x} + 3x^3)^3$

4. $y = (\frac{5}{x^2} + \frac{2}{x^3})^3$

5. $y = 3(4x^4 + 3x^2)^4$

6. $y = \frac{1}{2x+3}$

7. $y = \frac{5}{(5x^3 + 3x)^2}$

8. $y = \frac{6}{4(7x^2 + 3x - 4)^2}$

9. $y = \dfrac{2}{-(2x^2-7)^2}$

10. $y = \dfrac{-3}{8(9x^3+21x)^8}$

Product Rule

$y = uv$ u and v are functions in x

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Example

$y = (2x+3)(3x^3-2x^2-x)$

$u = 2x+3$ $v = 3x^3-2x^2-x$

$\dfrac{du}{dx} = 2$ $\dfrac{dv}{dx} = 9x^2-4x-1$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$=(3x^3-2x^2-x)(2)+(2x+3)(9x^2-4x-1)$$

Or differentiate directly

$y = (2x+3)(3x^3-2x^2-x)$

$$\frac{dy}{dx} = (3x^3-2x^2-x)(2)+(2x+3)(9x^2-4x-1)$$

Exercise 3

Find *f'(x)* of the following equation

1. $f(x) = x^2(2x+3)^3$

2. $f(x) = x^5(2x^3 + 3x)^2$

3. $f(x) = \frac{x}{5}(5x + 3x^2)^{-3}$

4. $f(x) = (x^2 - 12x)(5x + 11)^2$

5. $f(x) = (7x + 6)^2(3x^2 - 12)^4$

6. $f(x) = (5x + 1)^3 x^4$

Quotient Rule

$y = \frac{u}{v}$ u and v are functions in x

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Example

$$y = \frac{x^2}{2x+1}$$

$u = x^2$ $v = 2x+1$

$\frac{du}{dx} = 2x$ $\frac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2}$$

Or differentiate directly

$$y = \frac{x^2}{2x+1}$$

$$\frac{dy}{dx} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2}$$

Exercise 4

Find the first derivative of the following equation

1. $y = \frac{x+1}{x-1}$

2. $y = \frac{2x^2 - 2}{3x - 1}$

3. $y = \frac{2x}{2x^3 - 3x}$

4. $y = \frac{x+1}{3x-3x^2}$

6. $y = \frac{2-3x}{5(x^2-3x)}$

5. $y = \frac{1-2x}{2(x^3-3)}$

7. $y = \frac{(x-1)^3}{2-3x}$

8. $y = \frac{2x^2 + 3}{(3x - 12)^2}$

9. $y = \frac{7x + 1}{2x + 12} - \frac{5}{4x + 1}$

10. $y = (\frac{x}{2x - 1})^3$